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On the smallest screening masses in hot QCD

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ABSTRACT: The increasing focus on unquenched lattice simulations has revived interest also in gluonic screening masses, whose inverses characterise the longest length scales at which thermal fluctuations are correlated in a hot non-Abelian plasma. We fill an apparent gap in the literature concerning the theoretical structure of one of the relevant screening masses (the one which equals twice the Debye mass at leading order), by showing that the next-to-leading order correction to it is perturbative and small. This surprising result appears to explain semi-quantitatively why this particular channel yields the smallest gluonic screening mass at temperatures around a few hundred MeV (it couples to the energy density and to the real part of the Polyakov loop), even though it is not among the smallest screening masses at asymptotically high temperatures.

KEYWORDS: Thermal Field Theory, Lattice QCD, NLO Computations, Field Theories in Lower Dimensions

Contents

1	Introduction	1
2	General framework	2
3	Non-relativistic limit	3
4	Determination of M_2	4
5	Summary and conclusions	6
6	An open issue	7

1 Introduction

Screening masses, or inverses of equal-time correlation lengths, are a fundamental characteristic of the long-range properties of a thermal system. Indeed, the quantum numbers and the degeneracy of the excitation with the lowest screening mass indicate what kind of an effective theory it is that determines the infrared sensitive thermodynamic properties of the system, such as finite volume effects [1]. In QED, for instance, correlators of magnetic fields display a vanishing screening mass, while correlators of electric fields reveal a non-vanishing “Debye mass”; this then shows that at the longest length scales only magnetic fields are significant in an Abelian plasma, and finite-volume effects are powerlike.

In non-Abelian gauge theories such as QCD, it turns out that the situation with the screening masses is a bit more complicated than in QED. In fact, even the definition of what is meant by screening masses requires some care: electric and magnetic fields, on which our Abelian intuition is based, are no longer gauge-invariant objects. Because of these subtleties it was only in the mid-1990’s that fully satisfactory gauge-invariant and non-perturbative definitions were given to colour-electric, colour-magnetic, and certain more refined classes of screening masses in a non-Abelian plasma [2].

Following the conceptual clarification of the gauge-invariant definition of gluonic screening masses in QCD, systematic lattice measurements could also be carried out in all relevant channels. We would like to mention, in particular, quenched lattice measurements in four dimensions [3, 4]; unquenched lattice measurements via a dimensionally reduced effective field theory in three dimensions [5]; and, most recently, unquenched lattice measurements directly in four dimensions [6]. Of course, a systematic analysis of the same observables can also be carried out in the AdS/CFT framework [7].

The purpose of the present paper is to consider the screening masses within the weak-coupling expansion. A number of them fall into the general class of observables whose

leading-order value is fixed by the Debye scale; this class includes also many real-time observables of current interest, such as heavy-quark diffusion and jet quenching. It has been found in several such cases that the next-to-leading order correction is large for phenomenologically interesting values of the gauge coupling [8]. Our results will produce a “counter-example” to this empirical observation, showing that it is also possible to find observables in which the next-to-leading order correction is small.

2 General framework

In order to implement the resummations that are needed for defining the weak-coupling expansion at high temperatures, we choose to view the screening masses with the help of the dimensionally reduced effective field theory for hot QCD [9], called EQCD [10]. This approach is certainly sufficient for clarifying the theoretical structure of the various screening masses and, at least on the semi-quantitative level, also for numerical estimates. The effective Lagrangian has the form

$$\mathcal{L}_E = \frac{1}{2} \text{Tr} [F_{ij}^2] + \text{Tr} [D_i, A_0]^2 + m_E^2 \text{Tr} [A_0^2] + \dots \quad (2.1)$$

Here $F_{ij} = (i/g_E)[D_i, D_j]$, $D_i = \partial_i - ig_E A_i$, $A_i = A_i^a T^a$, $A_0 = A_0^a T^a$, and T^a are hermitean generators of SU(3). In three dimensions the dimensionality of g_E^2 is GeV. A 2-loop derivation of m_E^2 , g_E^2 in terms of the parameters of four-dimensional QCD can be found in ref. [11].

Correlation lengths are defined from the exponential fall-off of two-point functions of local gauge-invariant operators. Without a loss of generality we assume the two-point functions to be measured in the x_3 -direction. The independent channels can be classified according to discrete symmetries defined in the two-dimensional transverse (x_1, x_2) -plane. A particularly important symmetry is often called R, and corresponds to the CT-symmetry of the original QCD; in terms of eq. (2.1), it sets $A_0 \rightarrow -A_0$. “Colour-electric operators” are defined to be odd under this symmetry, while “colour-magnetic operators” are even [2].

With this notion, examples of operators from which colour-electric screening masses can be determined are $\text{Tr} [A_0 F_{12}]$ and $\text{Tr} [A_0^3]$. In four-dimensional QCD, these correspond to $\text{Im Tr} [P F_{12}]$ and $\text{Im Tr} [P]$, respectively, where P is the yet untraced Polyakov loop (we assume that the center symmetry is broken in the “trivial” direction, as is certainly the case in the unquenched theory). Note that these two channels do not couple to each other because of a different parity in the transverse plane. Typical operators from which colour-magnetic screening masses can be determined are $\text{Tr} [A_0^2]$ and $\text{Tr} [F_{12}^2]$, but any other gauge-invariant local singlet operator such as the energy density works as well. In four-dimensional QCD, $\text{Tr} [A_0^2]$ corresponds to $\text{Re Tr} [P]$.

It is important to note that in principle the operators $\text{Tr} [A_0^2]$ and $\text{Tr} [F_{12}^2]$ couple to each other [12, 13]. In other words, if we measure a correlation matrix between these operators, then the matrix includes non-diagonal components. It is possible, however, to diagonalize the correlation matrix at large distances, i.e. to find two orthogonal eigenstates which display different screening masses (see, e.g., refs. [14]). It is these eigenvalues of

the two-dimensional Hamiltonian that we refer to as M_2 and M_3 in the following. In practice, the coupling between $\text{Tr}[A_0^2]$ and $\text{Tr}[F_{12}^2]$ is very weak, both parametrically [13] and numerically [5], so it appears to us that it should play no actual role in our analysis.

Assuming that $m_E \gg g_E^2/\pi$, as is indeed the case at very high temperatures (in which limit $m_E \approx gT(N_c/3 + N_f/6)$, $g_E^2 \approx g^2T$, where $g^2/4\pi = \alpha_s$ is the strong gauge coupling, N_c is the number of colours, and N_f is the number of massless quark flavours), we can view A_0 as a heavy field and write down the parametric forms of various screening masses within the heavy-mass expansion.¹ In particular, the smallest screening mass in the colour-electric channel, coupling to $\text{Tr}[A_0 F_{12}]$, has a well-known logarithmic term at the next-to-leading order [15], and the general form [2]

$$M_1 \approx m_E + \frac{g_E^2 N_c}{4\pi} \left(\ln \frac{m_E}{g_E^2} + c_1 \right), \tag{2.2}$$

where $c_1 \approx 6.9$ for $N_c = 3$ [16]. This expression works reasonably well down to low temperatures, overestimating the “exact” value within EQCD by a modest amount [5]. In the colour-magnetic channel, we can expect the mass coupling to $\text{Tr}[A_0^2]$ to have, in the heavy-mass limit, the form

$$M_2 \approx 2m_E + \frac{g_E^2 N_c}{4\pi} \left(\ln \frac{m_E}{g_E^2} + c_2 \right). \tag{2.3}$$

Roughly, the correction here represents a three-dimensional bosonic analogue of the binding energy of a heavy quark-antiquark system, like J/ψ . As far as we can see it is a non-trivial fact, following from the analysis in section 4, that the coefficient of the logarithm in eq. (2.3) agrees with that in eq. (2.2). The colour-magnetic screening mass which is the smallest at asymptotically high temperatures can, in contrast, be obtained from the theory from which A_0 has been integrated out [13]; it couples dominantly to $\text{Tr}[F_{12}^2]$ and has the form

$$M_3 \approx \frac{g_E^2 N_c}{4\pi} \times c_3, \tag{2.4}$$

where $c_3 \approx 10.0$ for $N_c = 3$ [17].

3 Non-relativistic limit

Our goal now is to estimate the coefficient c_2 in eq. (2.3) which, to the best of our knowledge, remains unknown. The situation is quite similar to that in the case of the screening masses of fermionic bilinears, which we have studied previously in refs. [18, 19]. The two adjoint scalar fields form a bound state, and a formal scale hierarchy exists between the heavy scalar mass, m_E ; the relative momentum between the bound state constituents, $p \sim (g_E^2 m_E/\pi)^{1/2}$; and the binding energy, $\Delta E \sim g_E^2/\pi$, such that $p^2/m_E \sim \Delta E$ (logarithms and numerical factors have been omitted; note that in terms of the four-dimensional coupling the scales are

¹We stress that at this point the scale hierarchy $m_E \gg g_E^2/\pi$ serves only as a theoretical organizing principle for the computation; in practical estimates various group theory and numerical factors need to be added, and the phenomenological viability of the description can only be estimated *a posteriori*.

separated only by $\sim (g/\pi)^{1/2}$. This scale hierarchy can be employed for constructing a set of effective field theories, perhaps ultimately a scalar analogue of PNRQCD [20]. As argued in ref. [18], however, at the level of the correction of $\mathcal{O}(g_E^2/\pi)$, the whole procedure simply amounts to solving the Schrödinger equation in a two-dimensional Coulombic potential for the s-wave state; the only complication is that the heavy constituent “rest mass” entering the bound state problem needs to be fixed by a proper matching computation.

To nevertheless give a somewhat more concrete indication of the effective theory setup, let us carry out a Wick rotation from the 3-dimensional Euclidean theory to a (2+1)-dimensional Minkowskian theory, and rename the x_3 -coordinate to be time, t . Let us, furthermore, write the time dependence of the quadratic part of the action in Fourier space, with ω denoting the frequency:

$$S_E = \int_{\omega} \int_{\mathbf{x}} \text{Tr} \left\{ A_0(-\omega, \mathbf{x}) \left[-\omega^2 + m_E^2 - \nabla^2 \right] A_0(\omega, \mathbf{x}) \right\} + \dots \quad (3.1)$$

If we concentrate on modes close to the on-shell points, $|\omega \pm m_E| \sim g_E^2/\pi \ll m_E$, and write $\omega = m_E + \omega'$ or $\omega = -m_E - \omega'$, then we observe that the dynamics of the forward-propagating mode $A'_0(\omega', \mathbf{x}) \equiv A_0(m_E + \omega', \mathbf{x})$ and the backward-propagating mode $A_0^\dagger(\omega', \mathbf{x}) = A_0(-m_E - \omega', \mathbf{x})$ is determined by the non-relativistic Lagrangian

$$\mathcal{L}_E \approx 2m_E \text{Tr} \left\{ A_0^\dagger \left(-i\partial_t - \frac{\nabla^2}{2m_E} \right) A'_0 + A'_0 \left(i\partial_t - \frac{\nabla^2}{2m_E} \right) A_0^\dagger \right\}. \quad (3.2)$$

In configuration space, the original field A_0 is related to the new effective fields by $A_0 = e^{-im_E t} A'_0 + e^{im_E t} A_0^\dagger$. At leading order, then, the forward-propagating part of the composite operator $\text{Tr} [A_0^2(t)]$ has the energy eigenvalue $2m_E$.

When this argumentation is promoted to the quantum level, we expect the derivatives appearing in eq. (3.2) to get replaced by covariant derivatives, $\partial_t A'_0 \rightarrow [D_t, A'_0]$; the rapid oscillation frequency m_E to get replaced by a matching coefficient, which we denote by M_{rest} ; and the parameter m_E in the denominator of the kinetic term in eq. (3.2) to become another matching coefficient, which we denote by M_{kin} . In the limit $M_{\text{kin}} \rightarrow \infty$ the propagators of the A'_0 's are replaced by Wilson lines in the adjoint representation: $G(t, \mathbf{r}) \equiv \langle A_0^{a\prime}(t, \mathbf{r}) A_0^{a\prime}(t, \mathbf{0}) A_0^{b\prime}(0, \mathbf{r}) A_0^{b\prime}(0, \mathbf{0}) \rangle = \text{Tr} \{ U_{\text{adj}}(t, \mathbf{r}) U_{\text{adj}}^T(t, \mathbf{0}) \}$, where $U_{\text{adj}}(t, \mathbf{r})$ is a straight timelike adjoint Wilson line at spatial position \mathbf{r} and we have for brevity omitted the (non-unique) spacelike connectors that make the point-split operators gauge-invariant. The evaluation of this expectation value leads to the concept of a static potential in the usual way: $V(\mathbf{r}) = \lim_{t \rightarrow \infty} [i\partial_t G(t, \mathbf{r})] G^{-1}(t, \mathbf{r})$. For the actual bound state problem $M_{\text{kin}} \approx m_E$ stays finite and the static potential takes the role of a matching coefficient. We do not need to know more about the effective theory setup in the following but remark that a formal discussion can be found in ref. [22].

4 Determination of M_2

Proceeding now with the non-relativistic setup outlined above, we expect that in the heavy mass limit the bound state mass can be written as

$$M_2 \approx 2M_{\text{rest}} + \Delta E. \quad (4.1)$$

In dimensional regularization in $d = 3 - 2\epsilon$ spatial dimensions, the next-to-leading order value of the matching coefficient M_{rest} reads [2]

$$M_{\text{rest}} = m_{\text{E}} - \frac{g_{\text{E}}^2 N_{\text{c}}}{8\pi} \left(\frac{1}{\epsilon} + \ln \frac{\bar{\mu}^2}{4m_{\text{E}}^2} + 1 \right), \quad (4.2)$$

where $\bar{\mu}$ is the scale parameter of the $\overline{\text{MS}}$ scheme. The binding energy can be solved from a two-dimensional Schrödinger equation; the potential appearing in it, obtained by integrating out the time (or x_3) components of the gauge fields reads

$$V(\mathbf{r}) = g_{\text{E}}^2 N_{\text{c}} \int \frac{d^{2-2\epsilon} \mathbf{q}}{(2\pi)^{2-2\epsilon}} \frac{1 - e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2} = \frac{g_{\text{E}}^2 N_{\text{c}}}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\bar{\mu}^2 r^2}{4} + 2\gamma_{\text{E}} \right). \quad (4.3)$$

In total, then, we are looking for the ground state solution to the problem

$$\left[2M_{\text{rest}} - \frac{\nabla_{\mathbf{r}}^2}{m_{\text{E}}} + V(\mathbf{r}) \right] \Psi_0 = M_2 \Psi_0, \quad (4.4)$$

where the non-kinetic terms combine to the finite expression

$$2M_{\text{rest}} + V(\mathbf{r}) = 2m_{\text{E}} + \frac{g_{\text{E}}^2 N_{\text{c}}}{2\pi} \left[\ln(m_{\text{E}} r) + \gamma_{\text{E}} - \frac{1}{2} \right]. \quad (4.5)$$

In the kinetic term of eq. (4.4), we already expanded the (“reduced” version of the) matching coefficient M_{kin} to leading order in $g_{\text{E}}^2/\pi m_{\text{E}}$, as is sufficient at the current level of accuracy.

It is important to note that, unlike speculated in earlier works [21], no infrared divergences appear in eq. (4.5). The reason is that the logarithmic divergences originating from the “hard” momenta ($q \sim m_{\text{E}}$; eq. (4.2); viewed from this side $1/\epsilon$ is an infrared divergence) and the “soft” momenta ($q \sim 1/r$; eq. (4.3); viewed from this side $1/\epsilon$ is an ultraviolet divergence) of the spatial gluons A_i cancel against each other in eq. (4.5).

Carrying out suitable rescalings, eq. (4.4) can be solved up to one transcendental number. We thus obtain

$$M_2 \approx 2m_{\text{E}} + \frac{g_{\text{E}}^2 N_{\text{c}}}{2\pi} \left(0.60372466 - \frac{1}{2} \ln \rho \right), \quad (4.6)$$

where

$$\rho \equiv \frac{g_{\text{E}}^2 N_{\text{c}}}{2\pi m_{\text{E}}} = \frac{N_{\text{c}}}{2\pi y^{1/2}}. \quad (4.7)$$

At next-to-leading order in massless QCD the ratio $y \equiv m_{\text{E}}^2/g_{\text{E}}^4$ is renormalization group invariant [23], and can be written compactly as

$$y \approx \frac{(2N_{\text{c}} + N_{\text{f}})(11N_{\text{c}} - 2N_{\text{f}})}{144\pi^2} \left[\ln \frac{4\pi T}{\Lambda_{\overline{\text{MS}}}} - \gamma_{\text{E}} + \frac{4N_{\text{f}} \ln 2 - N_{\text{c}}}{11N_{\text{c}} - 2N_{\text{f}}} + \frac{5N_{\text{c}}^2 + N_{\text{f}}^2 + 9N_{\text{f}}/2N_{\text{c}}}{(2N_{\text{c}} + N_{\text{f}})(11N_{\text{c}} - 2N_{\text{f}})} \right]. \quad (4.8)$$

The corresponding $m_{\text{E}}/g_{\text{E}}^2 = y^{1/2}$ is plotted in figure 1 for $N_{\text{c}} = 3$.

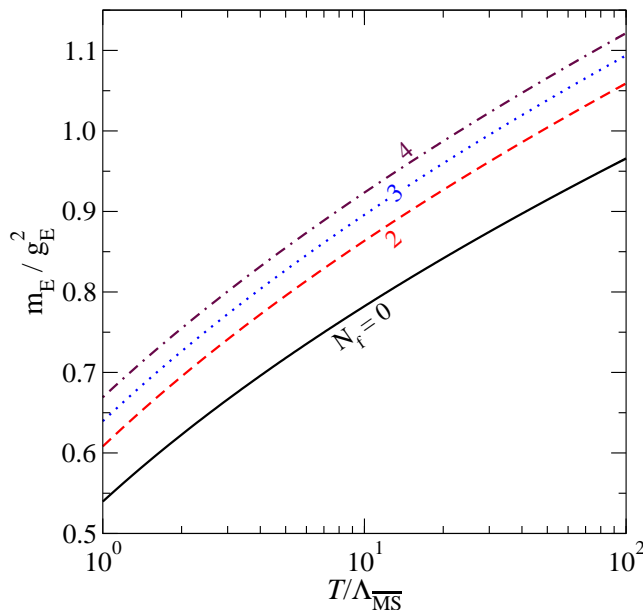


Figure 1. The parameter $m_E/g_E^2 = y^{1/2}$ from eq. (4.8) for $N_c = 3$.

5 Summary and conclusions

Comparing eq. (4.6) with eq. (2.3), we obtain

$$c_2 \approx 1.9467141 \tag{5.1}$$

for $N_c = 3$. Given that $m_E \geq 0.5g_E^2$ (cf. figure 1), the latter term in eq. (2.3) is always subdominant. This is in stark contrast to eq. (2.2), in which the latter term, containing the non-perturbative coefficient $c_1 \approx 6.9$, dominates in the whole temperature range of phenomenological interest. Note that because $c_2 \ll c_3 \approx 10.0$, M_2 is in general also below M_3 (cf. eq. (2.4)) in the temperature range of figure 1. All three masses are plotted in figure 2, both in units of g_E^2 and in units of T .

To summarize, it appears understandable that M_2 represents the smallest screening mass at realistic temperatures, because of the small perturbative coefficient $c_2 \approx 1.9$ in its next-to-leading order term, even though in the extreme limit $m_E \gg g_E^2/\pi$ it eventually overtakes both M_1 and M_3 , because a higher multiple of m_E 's appears in the leading term. This observation, together with the explicit results in figure 2, constitute the main points of this note.

We would like to stress, finally, that although our result for M_2 is not meant to be quantitatively accurate at low temperatures, it nevertheless reveals an interesting pattern. For example, for $N_f = 0$, figure 2 suggests $M_2/T \approx 3 \dots 4$ in the phenomenologically interesting temperature range, while lattice measurements indicate values $M_2/T \approx 2.5 \dots 3$ [3, 4], i.e. in the same ballpark but deviating downwards on the quantitative level. It appears, though, that this difference could at least partly be understood through higher order corrections *within* the EQCD effective theory defined by eq. (2.1): for $T/\Lambda_{\overline{\text{MS}}} \approx 2$, the non-perturbative

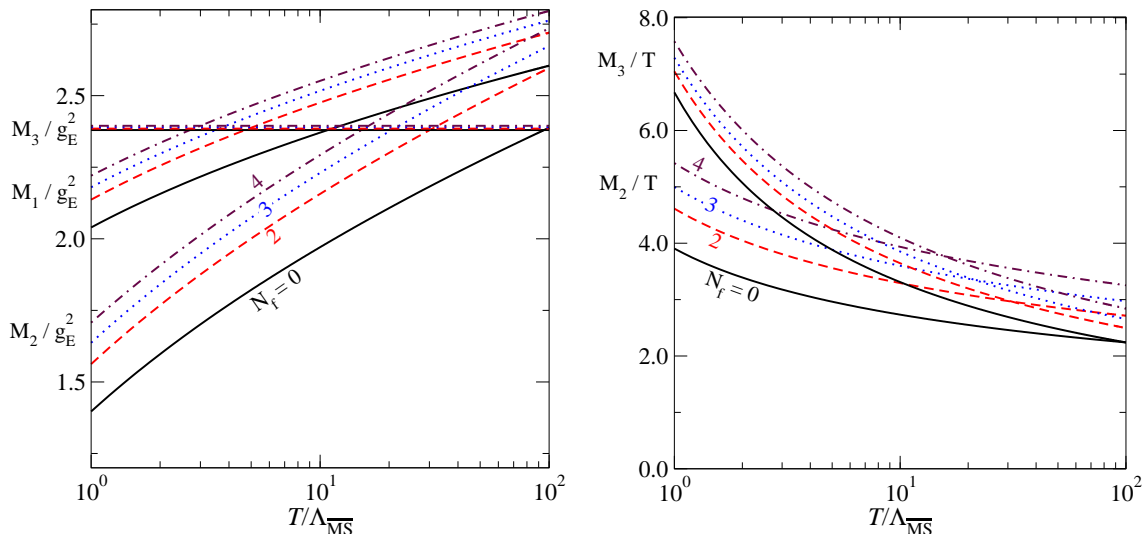


Figure 2. Left: The masses M_1, M_2, M_3 (cf. eqs. (2.2)–(2.4)), in units of g_E^2 , for $N_c = 3$, after insertion of m_E/g_E^2 from eq. (4.8). For better visibility, the four curves for M_3 , which are degenerate, have been slightly displaced from each other. Right: The masses M_2, M_3 , which belong to the same quantum number channel, after the insertion of the 2-loop value for g_E^2/T from ref. [11].

lattice measurements in ref. [5] yielded $M_2/g_E^2 \approx 1.0, 1.3, 1.5, 1.6$ for $N_f = 0, 2, 3, 4$, respectively, and $M_1/g_E^2 \approx 1.7, 2.0, 2.1, 2.1$ for the same cases; these values lie consistently somewhat below the perturbative estimates in the left panel of figure 2, resulting in a better accord with 4d lattice data. Moreover, a similar overshooting of the $\mathcal{O}(g_E^2)$ -corrected screening masses has been found for mesonic observables [18, 19]. So, it might be the general case that higher-order corrections, mostly from within three-dimensional EQCD dynamics, sum up to a negative correction to the next-to-leading order expression for screening masses. This would imply that screening masses are different in character from some dynamical quantities like the heavy quark momentum diffusion coefficient, in which case higher order corrections appear to *add up* on top of the already large next-to-leading order correction [24].

6 An open issue

We end by briefly pointing out an open problem to which we have no solution. Consider the screening mass extracted in four dimensions from the imaginary part of the Polyakov loop; in EQCD this corresponds to $M(\text{Tr}[A_0^3])$. In weak coupling, $M(\text{Tr}[A_0^3]) = 3m_E + \frac{g_E^2 N_c}{4\pi}(c_4 \ln \frac{m_E}{g_E} + c_5)$. Even though this is heavier than M_1 , particularly for $N_f > 0$ [5], it can be easily measured on the lattice [6], since the corresponding operators have different quantum numbers. Therefore, it would be nice to know the coefficients c_4, c_5 . Though this is certainly a well-defined problem (for $N_c > 2$), we have no clear idea about how it could be solved in a systematic way. (Probably the system can still be described by a non-relativistic many-body Schrödinger equation with a certain three-body potential in it, but in the absence of an effective theory framework or an explicit power-counting argument,

it is difficult to know for sure how to proceed without ambiguities.) If the pattern found in this note continues, however, we might expect c_4, c_5 to be coefficients at most of order unity, such that the leading-order term would dominate even more than in M_2 .

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